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(54) Measuring radar cross-section

(57) An arrangement (1) for determining the effective radar echo cross section of radar targets (2) as a function of the geometrical position of the target in relation to the radar antenna arrangement comprises radar antenna (11) mounted on a rotatable platform (10) outside the rotation axis (12) of the said platform (10). In order to enable the entire radar target (2) to be scanned in succession the platform (10) is pivotable about two axes (13, 14) perpendicular to the rotation axis (12). The radar target (2) to be examined is situated on a rotating stage (3) which rotates during the measuring operation. The arrangement can be used in cases in which the radar target (2) to be examined is no longer uniformly illuminated by the radar beam (because it is in the near field region or the target is large compared to the area illuminated) and the position of the phase centre (15) of the antenna (11) changes in relation to the target (2).

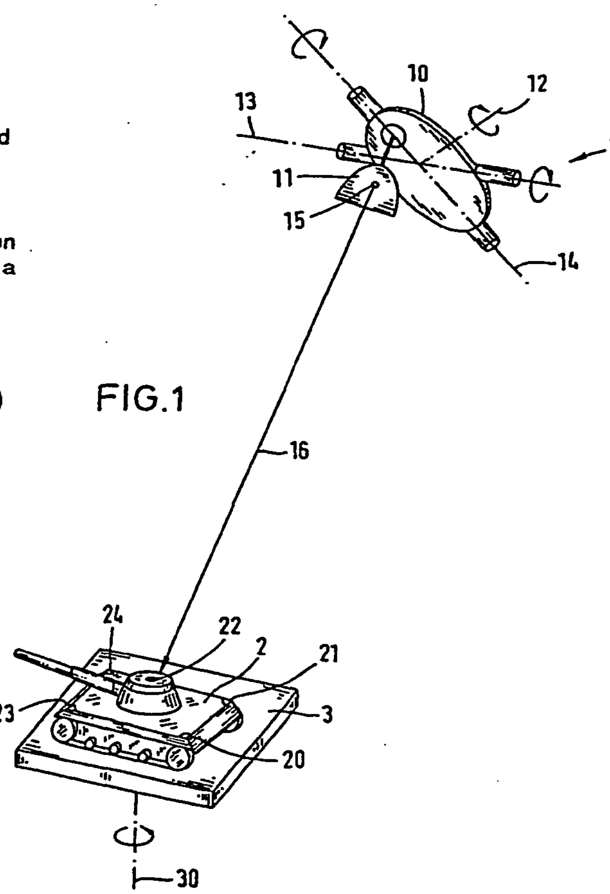

FIG.1

At least one drawing originally filed was informal and the print reproduced here is taken from a later filed formal copy.

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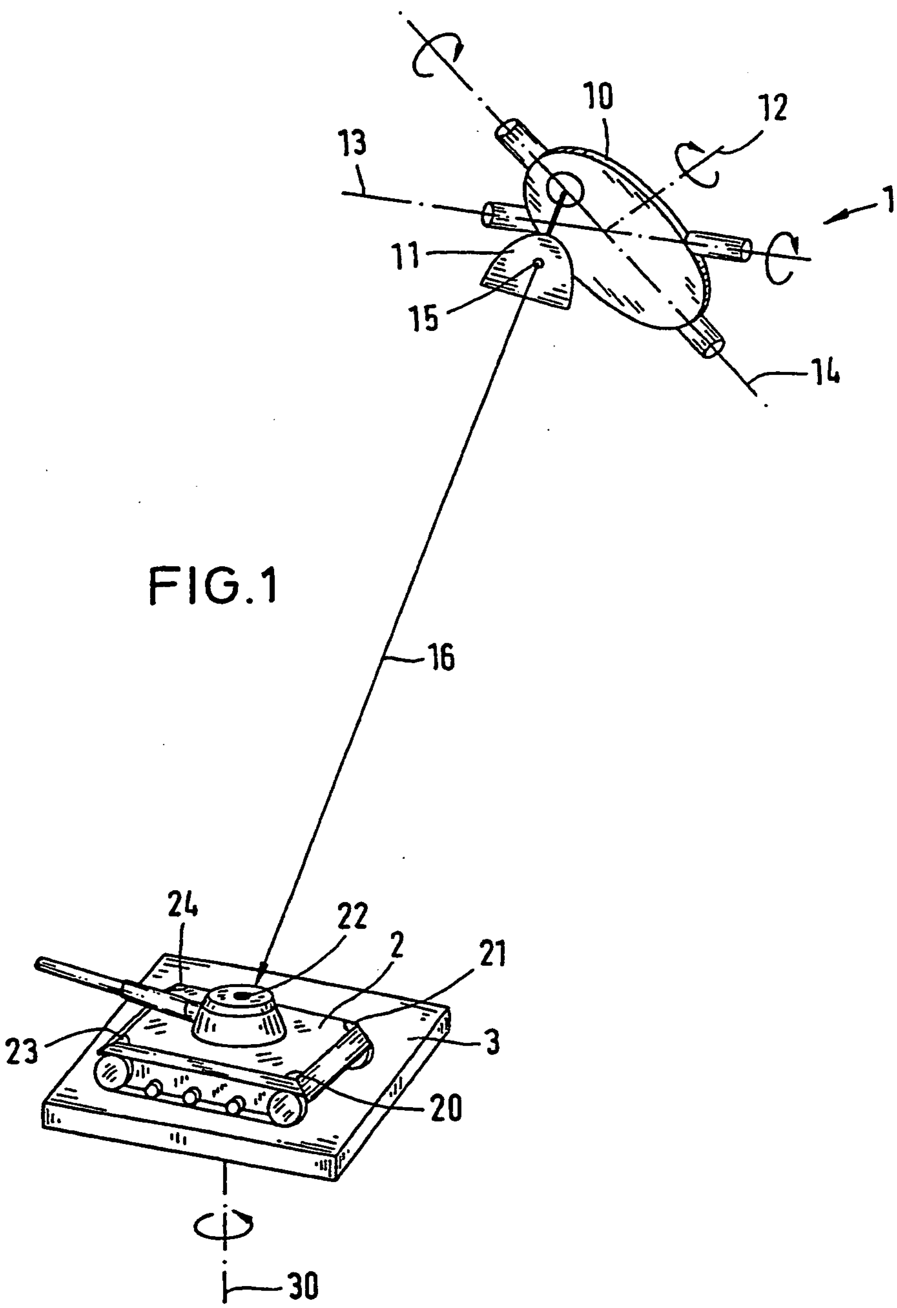

FIG.1

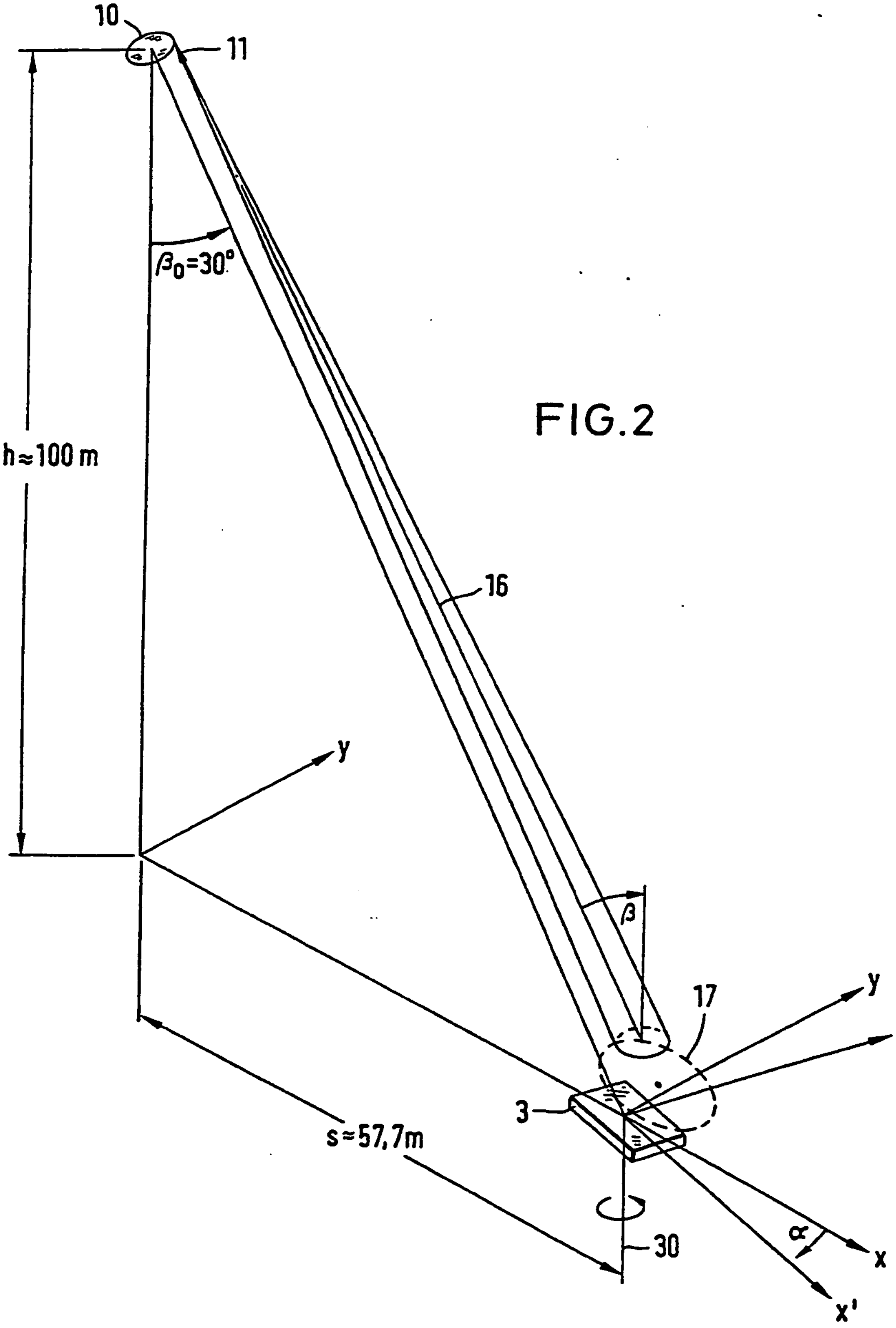

FIG.2

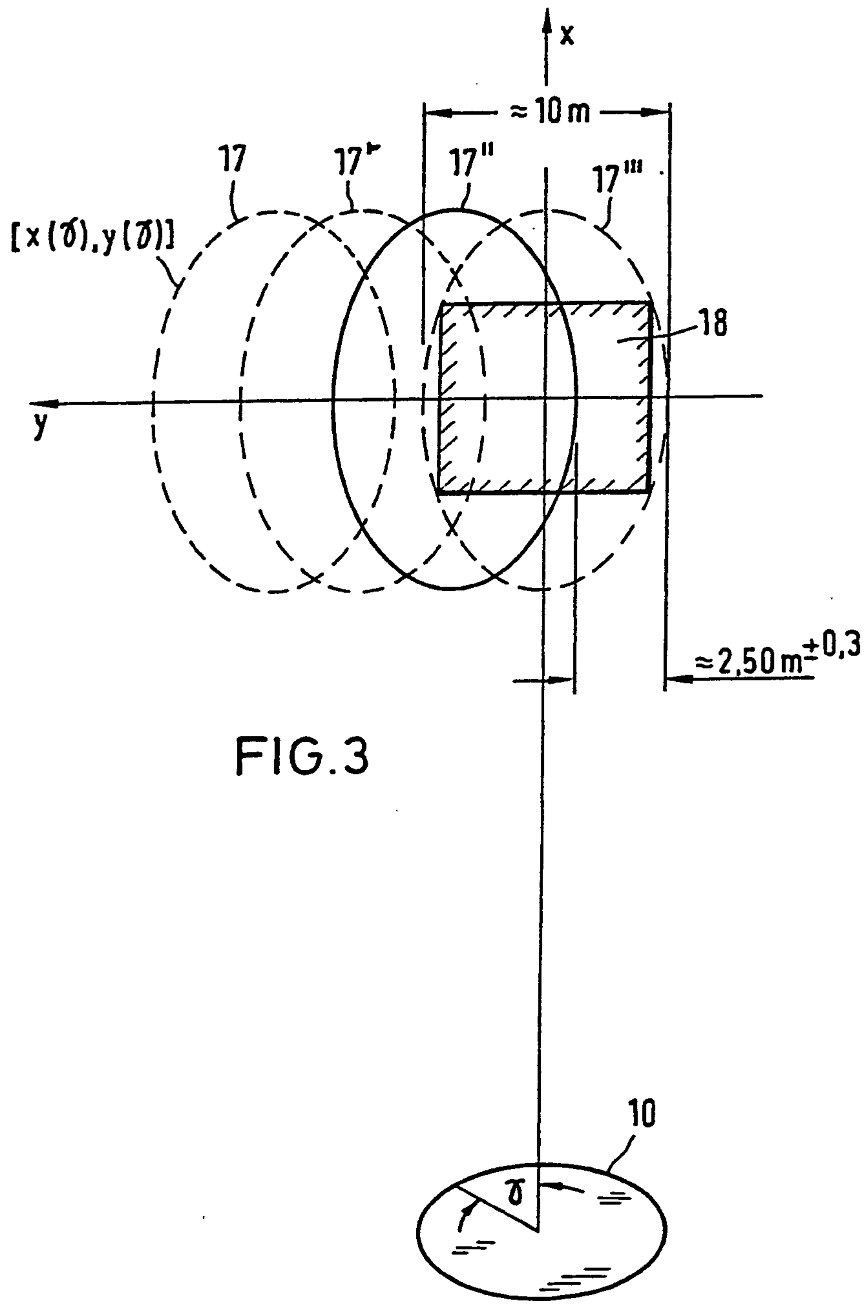

FIG.3

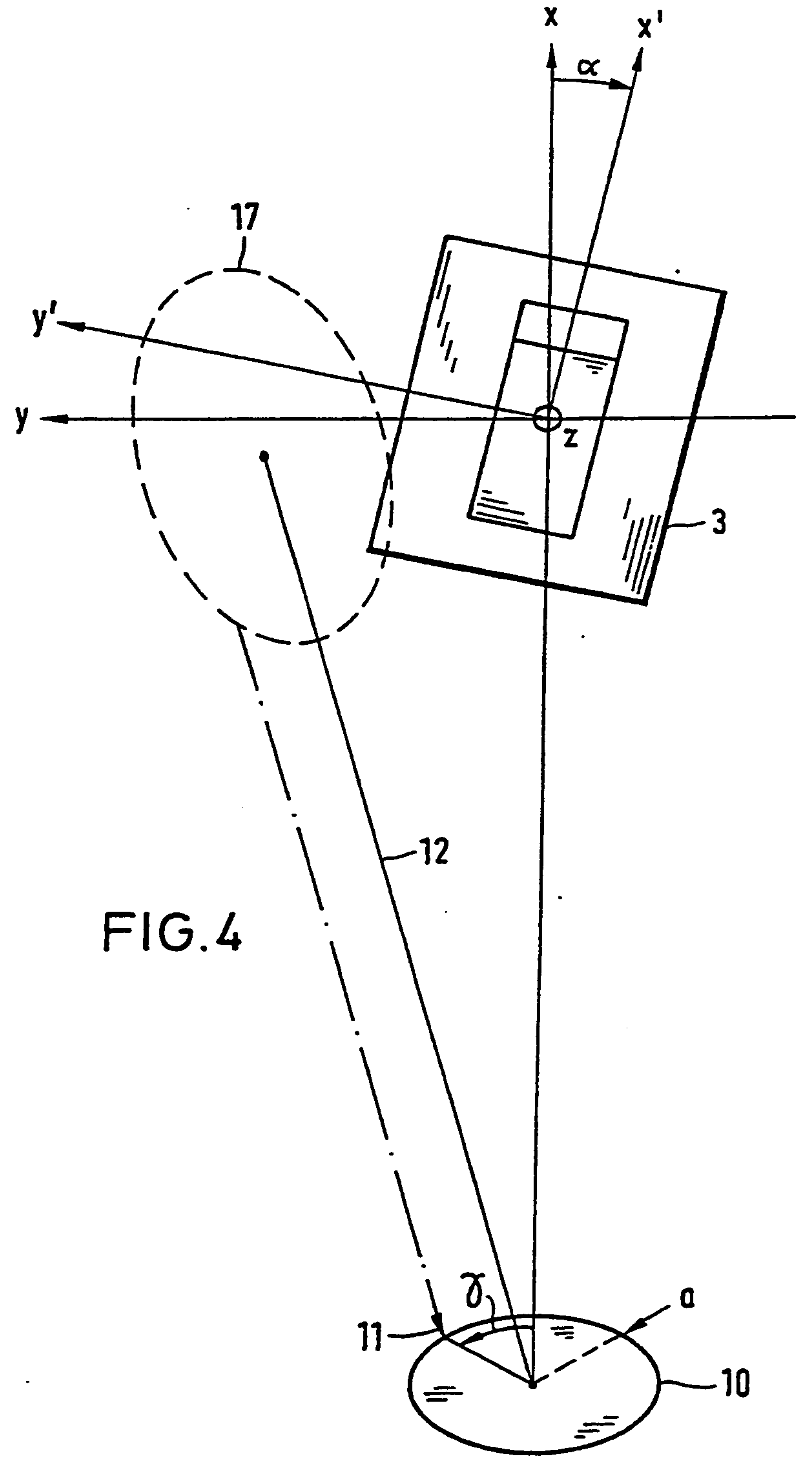

FIG.4

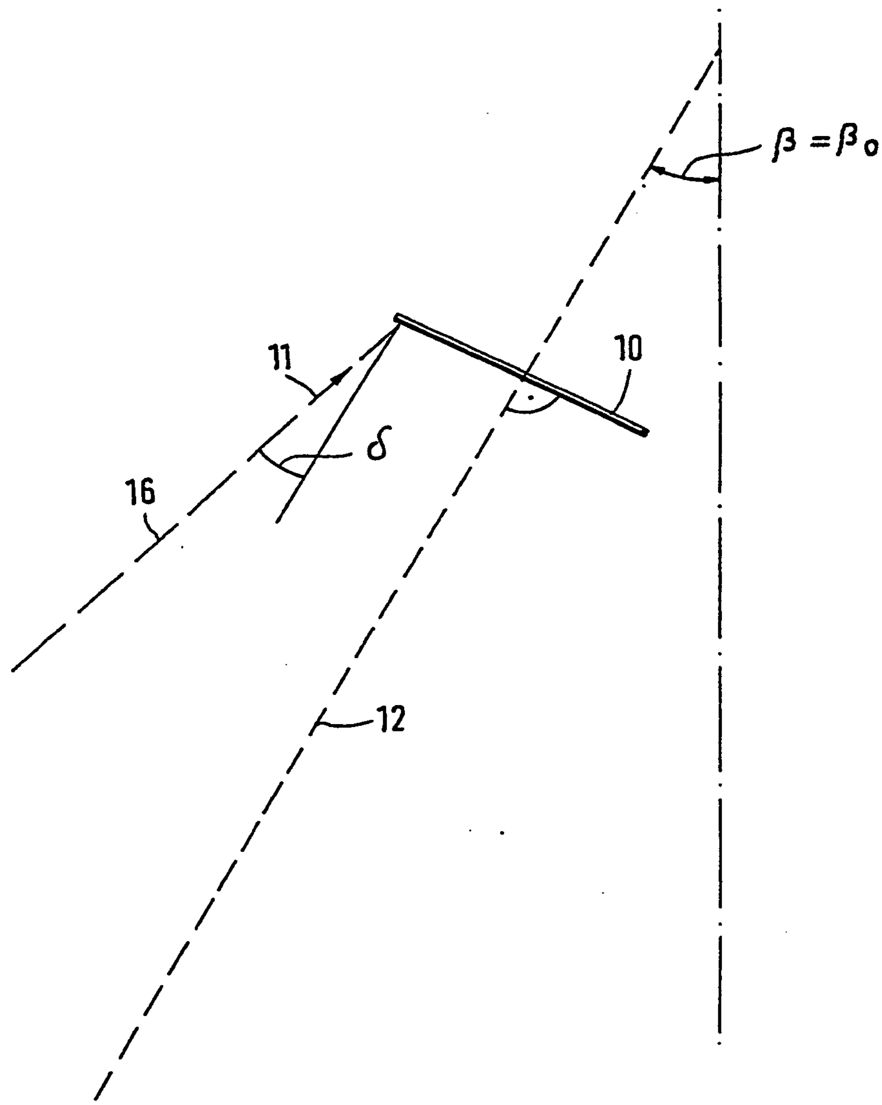

FIG.5

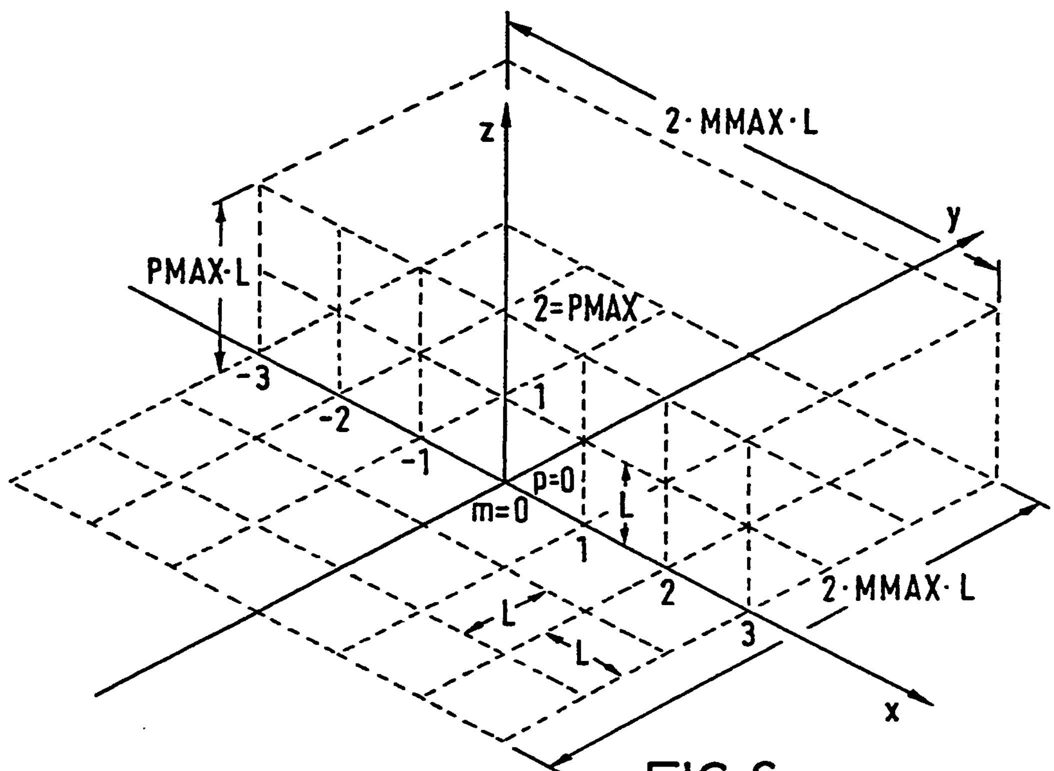

FIG.6

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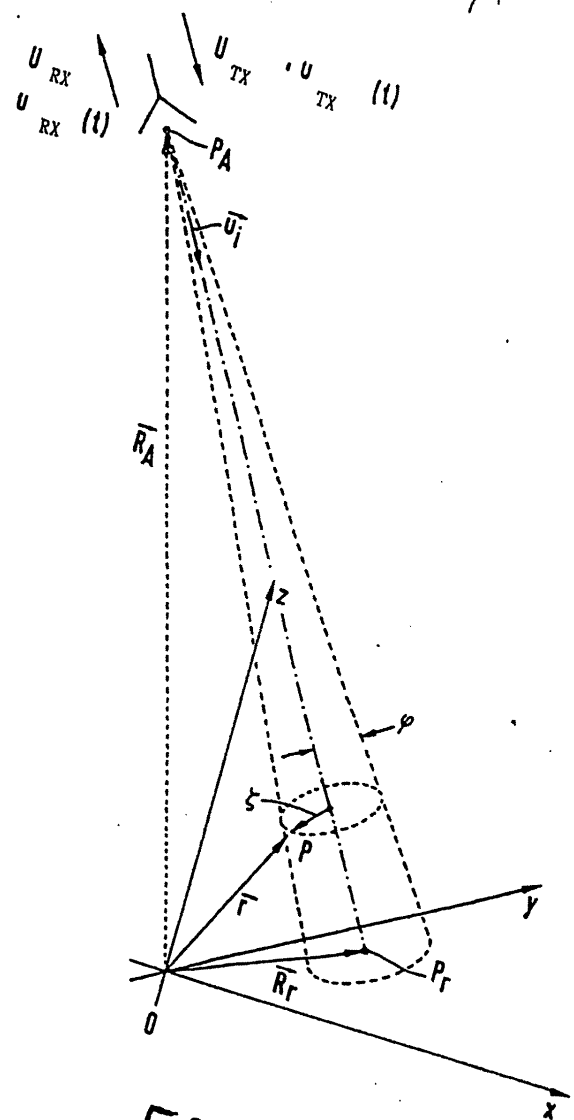
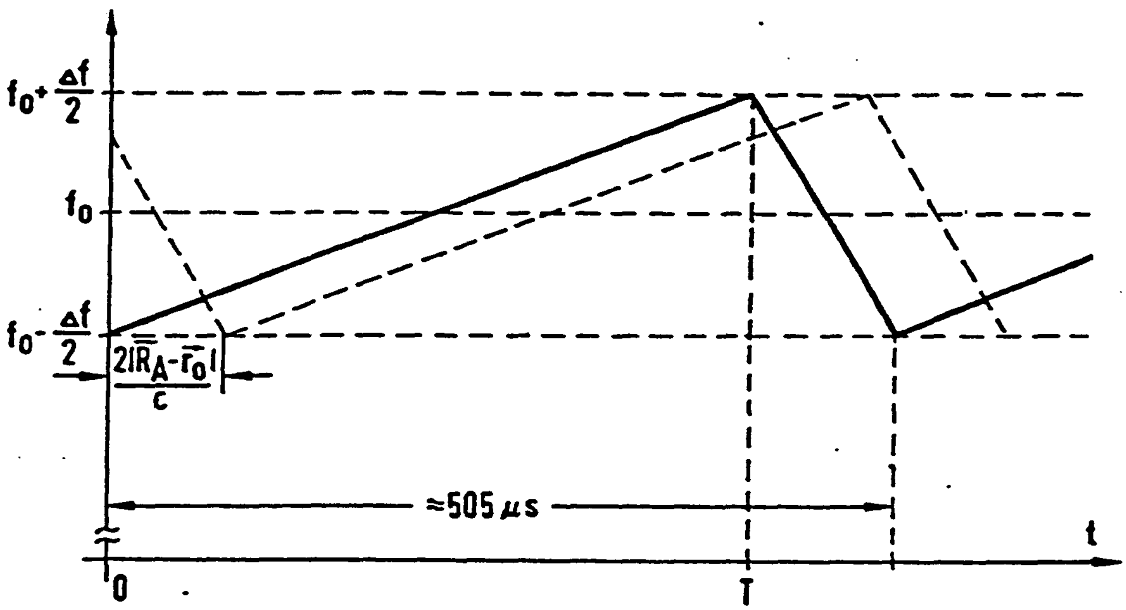


FIG 7



$f_0 = 94$ GHz $\Delta f = 120$ MHz

$T = 400\,\mu s$

$\delta_t = 0.8\,\mu s$

FIG 8

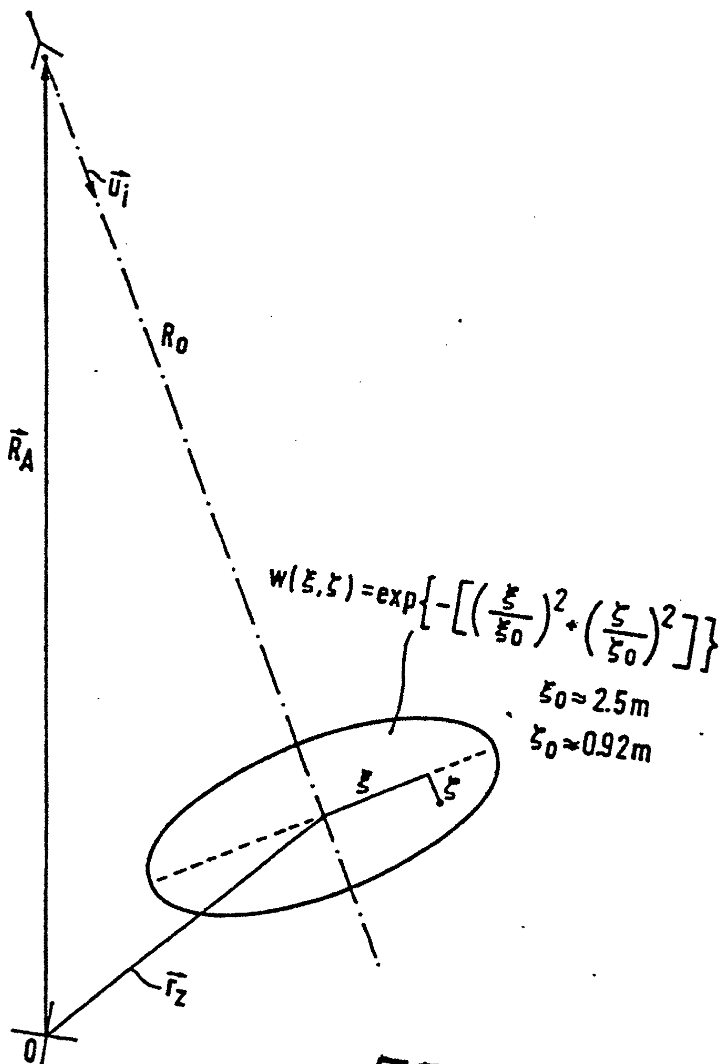


FIG 9

$\vec{r}_z = \vec{R}_A + \vec{u}_i \cdot v \cdot \delta_P$

(v: = NUMBER OF DISTANCE GATES

δ_P : = DISTANCE INCREMENT

TITLE

Radar Antenna

This invention relates to a radar antenna arrangement and to a method for determining the radar backscatter cross-section of a target.

To determine the behaviour of the radar echo cross section (σ) of a target as a function of the aspect angle the radar energy backscattered from the target is measured as a function of the aspect angle (for a given distance and a given angle of elevation). In this method, however, it is assumed that the target is completely illuminated and is situated remotely in the field of view of the receiving antenna.

It is for such cases that the term "radar backscatter cross section" is defined whereby the target acts as a single reflector (point disperser). If the extent of the target is appreciably greater than the wavelength of the radar then the backscatter cross section will be greatly dependent on the aspect angle. This dependence on the angle is unaffected by distance.

If the target is not fully illuminated by the radar beam (e.g. because the antenna has a concentrated beam) the target has to be scanned by the radar beam with only

a part of the target being illuminated during each scanning operation. The radar backscatter cross section measured in this case naturally then does not describe the radar backscatter characteristics of the entire target but only describes that part of the target which is illuminated by one half to the whole of the 3dB points in the antenna beam width. This is termed the effective radar cross section (σ eff). This expression is also used when the receiving antenna is located close to or in the "near field" of the illuminated target or illuminated portion thereof. By "near field" is meant the distance R for which the following equation applies:

$$R < \frac{2a^2}{\lambda}$$

wherein a = diameter of target and λ = wave length.

The knowledge of only one (σ eff) value, determined for a given position of the target in relation to the radar apparatus is insufficient as a means of determining (σ eff) values for other positions of target in relation to the radar apparatus, particularly in the event of variations in the distance or variations in the measurement unit (caused, for example, by the illumination of other parts of the target). A problem of this kind arises, for example, in determining the behaviour of a rotating seeker head for the detonator in

an ammunition unit descending by parachute and scanning a target area using millimetre wave length radar apparatus. In this case the phase centre of the antenna in space changes constantly on the one hand while the target is only partly illuminated on the other, owing to the narrow concentrated radar beam. This is not surprising as the wave length is usually far smaller than the dimensions of the target (such as a tank) and the receiving antenna is situated in the near field of the target.

One of the objects of this invention is to provide an apparatus having a means of determining the effectiv radar echo cross section if the radar target to be examined is no longer evenly illuminated by the radar beam and if the position of the phase centre of the antenna in relation to the target changes. Account may also be taken of cases in which the wave length of the radar is far smaller than the dimensions of the target.

A further object of this invention is to provide a method in which, on the basis of the values measured by the apparatus, a model can be constructed which will provide an effective echo cross section corresponding to the target, so that the behaviour of the target can eventually be simulated under various conditions.

These conditions include:

- Variable distance from radar to target at an elevation angle $\beta \sim \beta_o$.
- variable azimuth angle $0° \leq \alpha \leq 360°$
- variable antenna apex angle, (amongst others).

According to this invention there is provided a radar antenna arrangement for determining the effective radar echo cross section of radar targets as a function of the geometrical position of the target in relation to the radar antenna arrangement, wherein:-

(a) the radar antenna is mounted on a rotatable platform outside the rotation axis of the platform;
(b) the radar target to be examined is located on a rotating stage; and
(c) the platform is pivotable about two axes perpendicular to the rotation axis, so that the entire radar target can be successively scanned over a period of time.

According to this invention there is also provided a method for determining the effective radar backscatter cross section of a target using the radar arrangement wherein:

(a) the time dependent radar return signals are measured as a function of the position of the platform and the position of the rotating stage and subjected to a Fourier transformation;

(b) from the (discrete) spatial position of the resolution cell in each case and from the return signal data for an aspect angle range in each case the average value and the variance are determined;

(c) by allocating the average values and variances to the centre point of the resolution cell for each aspect angle range a three-dimensional image function of the average values and a three-dimensional image function of the variances are formed and are allocated to preselected geometrical positions for the scattering centres in order to indicate the backscatter characteristics of the target;

(d) the punctiform local radar cross sections are described for each aspect angle range by the average value of the radar cross section and its variance;

(e) from the average value and variance indications of the local radar cross sections and by using suitable weighting the average value and the variance of the effective radar cross section for a given resolution cell and a given aspect angle range are determined.

The invention is based on the reasoning that a radar target can be modelled by an assembly of I independent punctiform dispersion centres distributed over the geometric area of the target. The number of dispersed

points is to be kept to a minimum and the geometrical position will usually be selected to ensure that they will coincide with highly dispersive points on the target. The values of the radar backscatter cross sections of the individual "point dispersers" apply in each case to a fixed aspect angle (such as $\Delta\alpha = 3°$ at the azimuth) and this changes with an alteration into this zone. Altogether a target model will then in each case have 360 / 3 = 120 different aspect angle ranges (or 60, if the radar target has a plane of symmetry).

To enable the modelling conditions to be fulfilled, certain assumptions have to be made regarding the statistics of the way in which the radar section fluctuates according to the aspect angle. The statistical assumptions made depend on the ratio of the wave length to the type, magnitude and nature of the object. As a general principle it is assumed that within an aspect angle range the radar cross section σ_i of the i^{th} point disperser is made up by the superimposition of a coherent part σ_i^k and an incoherent (diffuse) part σ_i^{NK}. For the example explained in greater detail hereinafter the variation according to the aspect angle (in the azimuth over the range of 3° for example, and with a sufficient variation of the angle of elevation) is considered, over

the distribution function, to be as follows

$$P(\sigma_i) = \frac{I}{\langle\sigma_i^K\rangle} \exp\left[- \frac{\sigma_i - \sigma_i^{NK}}{\langle\sigma_i^K\rangle} \right]$$

The diffuse part is considered to be constant within an aspect angle range, which in the present example is $\Delta\alpha = 3°$. The scatter of a point disperser for an aspect angle range is thus described completely by two real parameters, being the average value of the coherent part (σ_i^K) and the incoherent part σ_i^{NK}. The variance of σ_i with σ_i^K is thus expressed by:

$$\langle (\sigma_i - \langle\sigma_i\rangle)^2 \rangle = \langle \sigma_i^K \rangle^2$$

The mathematical discussion of the effective radar backscatter cross section set forth in the Appendices 1.3 and 1.4.

A complete description of the backscatter over all aspect angle ranges thus necessitates

$$I \times 120 \times 2$$

real parameter values. In the event of a plane of symmetry the above number is halved.

This invention is further described and illustrated with reference to an embodiment shown as an example and described with reference to the drawings, wherein:

Figure 1 shows a schematic diagram, in perspective, of an example for determining the

effective radar echo cross section of a tank using a radar antenna arrangement which simulates the behaviour of a missile approaching the target,

Figures 2 to 6 show different schematic representations serving to clarify the method. and

Figures 7 to 9, illustrate the discussion set out in the Appendices.

As shown in Figure 1, 1 is a radar arrangement, 2 a target (in the present case a tank) and 3 a rotating platform. The radar target 2 is situated on the platform 3 which turns about axis 30 during the method of determining the effective radar cross section. The radar antenna arrangement 1 mainly comprises an antenna platform 10, to which the radar antenna 11 is affixed.

The antenna rotates about axis 12 of the platform 10 which has two further axes of rotation 13 and 14. The phase centre (focal point) of the antenna 11 is marked 15 and the axis of the antenna lobe being marked 16.

As may be seen from Figure 1, the antenna 11 is positioned outside the rotation axis of the antenna platform 10. The invention thereby offers the advantage that on the rotation of the antenna platform 10 about the axis 12 the distance between the radar antenna 11 and the target 2 changes continuously. This provides an approximate simulation of the measurement of the

backscatter cross section by the radar arrangement of a missile approaching the target.

The measuring process to which the invention relates will be described below in greater detail:

As previously mentioned herein, the method of this invention is based on the principle of substituting the individual scatter centres (local radar cross sections) for the backscatter of the radar target in the event of partial illumination of the target and if the radar wave length is appreciably smaller than the dimensions of the target. Figure 1 shows five examples of scatter centres for this purpose referenced 20 to 24. In order to determine the value for the radar backscatter cross sections of these locally preselected reflectors, the target 2 is scanned by changing the inclination of the antenna platform 10 by rotation about the axes 13 and 14 in succession and recording the echo data as a function of the position of the antenna platform and that of the rotating stage for each angle of inclination with the antenna platform rotating about the axis 12. In a practical example the echo data were measured for four fixed positions (angles of inclination) of the antenna platform.

The radar apparatus used in the example had the following specifications:

<u>FMCW Type</u>

Basic frequency:	94 GHz.
Shift:	120 MHz.
Pulse rise:	400 μs.
Pulse fall:	100 μs.
Half power points of antenna:	appr. 2°

The mathematical description of the echo signal for this radar apparatus is given in Appendix 1.1. The measuring data were recorded from a tower with a height (h) of 100 m; the angle of inclination amounted to about 30°.

The sweep of the antenna lobe over the ground must be known and it has to be measured. The course taken by the sweep was determined, in the example, with an accuracy of $\pm$ 0.2 mm by the aid of a laser measuring device and/or with displaceable radar reflectors. In this process a table has to be drawn up for each of the four positions, showing the unambiguous interrelationship between the angle γ provided by an angle transmitter of the antenna platform and the sweep co-ordinates x (γ), y (γ) and z (γ). In this connection it is sufficient to measure about eight points per ellipse. The others can be determined therefrom arithmetically.

For the antenna platform angle γ pairs of angles should be selected which in each case differ by π (180°)

Figure 2 shows the record of the echo data for a certain inclination of the antenna platform. The item marked 17 is the sweep of the axis of the antenna lobe. Figure 3 shows a plan view of the measuring system from which the sweeps 17 to 17''' of the axes of the antenna lobes for the different positions of the antenna platform 10 may be determined. Figure 4 gives a corresponding plan view of the measuring system in the measurement of the sweep 17 with the rotary stage rotated. Figure 5 shows in lateral view the antenna 10 on the antenna platform 10, the angle of inclination of the antenna being marked σ .

With each of the four positions the echo data are recorded with the antenna platform rotating (platform angle γ with 3 x 360°/sec). In the example the rotation angle α of the rotary stage 3 changes at the rate of 360°/5 min. In this process five hundred scanning voltage values are read per echo and about six hundred and sixty six echoes measured per rotation of the antenna platform. For each echo the relevant platform angle γ is read out.

For each of the four fixed positions of the antenna platform 10 a data block is obtained. For each of the

four data blocks there is a corresponding table with the sweep data in accordance with the table below.

Object angle	Sensor angle	ZF voltage values
$\alpha_0 - \Delta\alpha/2$	γ_1	$u_1(t1)$ $u_1(t2)$....$u_1(t500)$
...	...	...
...	...	...
...	...	...
...	...	...
...	...	...
$\alpha_0 + \Delta\alpha$ 2	γ_N	$u_N(t1)$, $u_N(t2)$.....$u_N(t500)$

In the data block echo data are stored one after the other. They consist in each case of the two angles γ (rotation angle of the antenna platform) and α (rotation angle of rotary stage) and five hundred real scanning values $u(t_i)$ of the echo voltage. For each rotation of the antenna platform about six hundred and sixty six echoes are obtained. During a rotation of the antenna platform the object stage rotates by about 0.4° (in the example considered). We thus obtain the data for an aspect angle range of the width of $\Delta\alpha = 3°$ from 7.5 x 666 = 4995 echoes. If we wish to cover all aspect angle ranges from 0° to 360°, therefore, 120 x 5000 = appr. 600,000 echoes have to be evaluated. The evaluation of these data and the calculation of the corresponding local radar cross sections for the points

20 to 24 (Figure 1) are carried out using a four-dimensional image function and a subsequent expanded algorithm.

Calculation of four-dimensional image function.

For every data block of raw data, that is for every fixed position of the antenna platform, there is a special change in the orientation u^i (unity vector) of the antenna lobe with the platform angle γ. This change can be described by the equation

$$\vec{ui}\ (\gamma) = \vec{c} + \vec{a} \cos (\gamma) + \vec{b} \sin (\gamma)$$

wherein $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually orthogonal vectors.

These vectors can be determined from the measured points $\vec{r}$ (γ k) through which the axis of the radiation lobe passes at γ = γ k. In this process the measuring point γ k must also include the measuring point γ k + 180°.

Each echo signal is represented by the angle γ and α and the five hundred real-value scanning values from u_1 to u_{500}. To effect a 512-point FFT (Fast Fourier Transformation) the last twelve positions are occupied with noughts. With a 1024 FFT, 524 zeros are thus read in after the measuring data.

As a result of the FFT we obtain 512 and 1024 complex numbers U_m. The m^{th} number U_m belongs to a

distance gate of the distance $R_m = m \, \delta R$.

This provides the distance increment as follows:

$$\delta R = \frac{c \cdot T}{2} \frac{\delta f}{\Delta f}$$

wherein c = speed of light,
Δf = frequency sweep,
T + rise time,
δt = scanning rate.

and

$$\delta f = \frac{1}{NFFT \cdot \delta t}$$

In the above, Δf = 120mHz, T = 400 μs, δt = 0.8 μ s. We thus have, for the distance increment $\delta R(m)$ = 625/NFFT (NFFT = number of support points).

This provides the central point of the resolution cell belonging to the angle γ and the number (m) of the distance gate, as follows:

$$\vec{r}_M = -s\vec{u}_x + h\vec{u}_z + m\delta R \cdot \vec{u}_i(\gamma)$$

wherein s = distance of rotary stage,
h = height of head

(In the above: s(m) = 57.7; h(m) = 100).

(For a definition of the term resolution cell in conjunction with the discrete Fourier transformation, see Appendix 1:2.)

We thus have, for the Cartesian components of $\vec{r}_M$;

$$x_M = -s + m \cdot \delta R \, (c_x + a_x \cos(\gamma) + b_x \sin(\gamma))$$

$$y_M = m \cdot \delta R \, (c_y + a_y \cos(\gamma) + b_z \sin(\gamma))$$

$$z_M = h + m \cdot \delta R \, (c_z + a_z \cos(\gamma) + b_z \sin(\gamma))$$

wherein ax, ay, az, bx denote the Cartesian components of the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

As only some of the distance gates ($m = 1$ to 512) belong to distances corresponding to that of the object, only the gates with the numbers m_{min} to $m_{min} + 9$ are read out.

If, for a fixed aspect angle α, we enter the values of $|U_m|^2$ (m^{th} value of the discrete spectrum of the ZF signal) as a function of $\vec{r}_M$, we obtain a real-value image function

$$\tilde{F}(\alpha, \vec{r}_m)$$

As F is dependent on α and on the 3 co-ordinates of $\vec{r}_M$, it is designated a four-dimensional image function.

The description will now be given of an algorithm with which a discrete and averaged image function

$$F(\alpha_\mu, \vec{r}_M)$$

can be calculated for aspect angle ranges

$$\alpha_\mu - \frac{\Delta\alpha}{2} \leq \alpha \leq \alpha_\mu + \frac{\Delta\alpha}{2}$$

wherein $\Delta\alpha = 3°$. For this purpose, and in accordance with Figure 6, a three-dimensional grid with the grid width L(in (m)) is defined.

The grid points are indicated by

$$\vec{r}_{m,n,p} = L\,[m \cdot \vec{u}_x + \vec{u}_y + p \cdot \vec{u}_z];$$

wherein
L = grid distance,
m,n,p = integers

the integers m and n ranging from -MMAX to +MMAX and p from 0 to PMAX.

There are thus a total of $(2\ MMAX + 1)^2 \times (PMAX + 1)$ grid points.
(Maximum value: $MMAX \leq 6$, $PMAX \leq 3$).

There thus cannot be more than $13^2 . 4 = 676$ grid points for each of the 60 aspect angle ranges.

A measuring value is allocated to the grid point nearest to the relevant central point of the resolution cell. If in the course of the process this angle is encountered again (possible a number of times) in the same aspect angle range, then an average value $\vec{F}_{m n p}$ and a variance value $S_{m n p}$ is formed as follows:

$$F(m,n,p) = \frac{1}{I} \sum_{i=1}^{I} F_i(m,n,p)$$

and

$$S(m,n,p) = \frac{1}{I} \sum_{i=1}^{I} \left(F_i(m,n,p) - F(m,n,p) \right)^2.$$

To form these expressions it is in each case necessary to store the last values of $\vec{F}$ and S and also I (see Appendix 2).

The grid point (m,n,p) belonging to the co-ordinates (x_m, y_M, z_M) of the central point of the resolution cells calculated by rounding up or rounding down the co-ordinates scaled to L:

m = integer - minimum distance of $\left\{\frac{x_M}{L}\right\}$

n = integer - minimum distance of $\left\{\frac{y_M}{L}\right\}$

p = integer - minimum distance of $\left\{\frac{x_M}{L}\right\}$

(x_M, y_M, z_M: Cartesian components of vector $\vec{r}_M$).

If the magnitude of $|m|$ or $|n|$ exceeds the preselected value MMAX, the measuring value will not be allocated to any grid point. The same applies if p is either negative or greater than PMAX.

<u>The expanded algorithm</u> (cf.also Fig. 7)

The image function F_j = F (m,n,p) calculated for each aspect angle range $\alpha_V - \Delta\alpha/2 < \alpha < \alpha_V + \Delta\alpha/2$ in the fixed grid points $\vec{r}_j = \vec{r}_{m(j).n(j).p(j)}$ is obtained by combining the spatial distribution of the radar cross section $\sigma(\vec{r})$ with the weight function $w(\vec{r} - \vec{r}_j)$. In this process the weight function is formed as the product of the (transverse) function approximating the directional characteristic of the antenna

$$w_{tr}^2\left[\xi(\vec{r} - \vec{r}_j)\right] = \exp\left[-2\left(\frac{\xi}{\xi_0}\right)^2\right] \qquad (1)$$

with ξ = 2.5 m

and the longitudinal weight function

$$w_l^2\left[\zeta(\vec{r}-\vec{r}_j)\right] = \exp\left[-2\left(\frac{\zeta}{\zeta_0}\right)^2\right] \qquad (2)$$

approximating the characteristic of the distance gate (with ζ_0 = 0.92 m):

$w^2 = w_l^2 \cdot w_{tr}^2$

ζ is the longitudinal distance of the upper point $\vec{r}$ from the central point of the resolution cell $\vec{r}_j$ and is calculated as

$$\zeta(\vec{r}-\vec{r}_j) = |(\vec{r}-\vec{r}_j)\cdot\vec{u}_l|. \qquad (3)$$

With $\vec{u}_l = \frac{1}{2}\left(\vec{u}_x - \sqrt{3}\,\vec{u}_z\right)$

it follows that

$$\zeta = \left|\frac{1}{2}(x - x_j) - \frac{\sqrt{3}}{2}(z - z_j)\right|. \qquad (4)$$

The transverse distance ξ is calculated as

$$\xi = \sqrt{(x - x_j)^2 + (y - y_j)^2 + (z - z_j)^2 - \zeta^2} \qquad (5)$$

The spatial distribution of the radar cross section can be assumed in the form of I point reflectors having radar cross sections σ_l (l = 1.......I). The co-ordinates x_l', y_l', z_l' of these points in the system of co-ordinates rigidly connected with the object stage can be freely preselected. They should nevertheless be

positioned as far as possible in the vicinity of localities in which dispersion centres are situated.

For given angles the point reflector co-ordinates can be converted from the swept co-ordinate system (i.e. that rigidly connected with the object stage) to the spatially fixed co-ordinate system:

$$\begin{aligned} x_i &= x_i' \cos(\alpha) + y_i' \sin(\alpha) \\ y_i &= -x_i' \sin(\alpha) + y_i' \cos(\alpha) \\ z_i &= z_i' \end{aligned} \tag{6}$$

The combining in the discrete form, according to

$$F_j = \sum_{i=1}^{I} A_{ji}\, o_i \qquad , j = 1, \ldots, J \tag{7}$$

is thus found to be

$$A_{ji} = w_l^2\left[\zeta(\bar{r}_i - \bar{r}_j)\right] w_{tr}^2\left[\xi(\bar{r}_i - \bar{r}_j)\right]. \tag{8}$$

As the number J of grid points in which the image function was calculated will be higher than the number I of the point reflectors Eq. (7) represents a redundant equation system. Here the solution selected for can be that leading to the minimum quadratic error:

$$\sum_{j=1}^{J} |\tilde{F}_j - F_j|^2$$

An analogous argument is effected for the variance.

For the scaling of the "statistical" backscatter cross sections thus obtained a calibration measurement is required (see App. 3)

APPENDICES

Appendix 1:1

Echo signal of CW-FM Radar

In this section the interrelationship between the data of the echo signal in the NF part of the CW-FM radar and the spatial distribution of the radar cross section with any antenna position and antenna orientation is to be determined.

For this purpose an unambiguous description of the antenna position and orientation will first be introduced by means of the so-called "antenna parameters". According to Figure 7 the zero point 0 of the co-ordinate system is situated in the rotation centre of the object stage. The phase centre of the antenna in the point P_A described by the position vector

$$\vec{R}_A = x_A \vec{u}_x + y_A \vec{u}_y + z_A \vec{u}_z$$

$\vec{u}_x, \vec{u}_y, \vec{u}_z$: unit vector in X, Y, Z direction

x_A, Y_A, Z_A: Cartesian co-ordinates

Let the axis Γ of the rotationally symmetrical antenna lobe point in the direction of the unit vector $\vec{u}_1$ (cf. Fig. 7)

If some point P_Γ (other than P_A) on the axis Γ is known, the $\vec{u}_1$ can be calculated by the formula

$$\vec{u}_1 = \frac{\vec{R}_\Gamma - \vec{R}_A}{|\vec{R}_\Gamma - \vec{R}_A|} \qquad (1.1.1.)$$

If the effective power $p^{(S)}$ is fed into the antenna, then the corresponding electromagnetic field E in the range

$$|\vec{r} - \vec{R}_A| > \frac{2 D_A^2}{\lambda_0} \qquad (1.1.2)$$

(distant field condition for antenna field, D_A: antenna diameter)

is found to be as follows:

$$\vec{E}(\vec{r}) = \frac{\exp\left(-j2\pi \frac{|\vec{r} - \vec{R}_A|}{\lambda_0}\right)}{|\vec{r} - \vec{R}_A|} \sqrt{\frac{Z_0}{2\pi}} \; C(\varphi) \cdot \sqrt{P^{(S)}} \; \vec{e} \qquad (1.1.3)$$

$(Z_0 = 377\,\Omega)$.

In the above the unit vector $\vec{e}$ situated perpendicularly on $(\vec{r} - \vec{R}_A)$ characterises the polarisation, while C (φ) is the absolute directional characteristic. C (φ) is defined in such a way that the square of its magnitude is equal to the gain function $\tilde{G}$:

$$\tilde{G}(\varphi) = C^2(\varphi) \qquad (1.1.4)$$

Thus C^2 (φ) is equal to the gain G of the antenna.

In order, in Eq. (1.1.4), to express the angle φ by the upper point vector $\vec{r}$, the distance of the said vector F from the axis Γ is first of all determined (see Figure 7). We thus have:

$$\zeta = \left|(\bar{r} - \bar{R}_A) - [(\bar{r} - \bar{R}_A) \cdot \bar{u}_1]\bar{u}_1\right| \tag{1.1.5}$$

and thence:

$$\varphi(\bar{r}) = \arcsin\left(\frac{\zeta}{|\bar{r} - \bar{R}_A|}\right). \tag{1.1.6}$$

If in the position $\vec{r}_o$ there is a "point disperser" with the radar cross section * σ and the scatter phase Ψ then the reception voltage U_{Rx} at the antenna gate is obtained as a function of the transmission voltage U_{Tx} at the same gate, as follows:

$$U_{Rx} = \frac{\lambda_0 e^{-j\left(\frac{4\pi}{\lambda_0}|\bar{R}_A - \bar{r}_0| + \Psi\right)}}{(4\pi)^{3/2}\sqrt{L_C}\,|\bar{R}_A - \bar{r}_0|^2} C^2(\varphi)\sqrt{\sigma} \cdot U_{Tx} \tag{1.1.7}$$

In the above L_c is the loss factor.

With N different reflectors with the radar cross section σ_n and phases Ψ_n we thus obtain

$$\frac{U_{Rx}}{U_{Tx}} = \frac{\lambda_0}{(4\pi)^{3/2}\sqrt{L_C}} \sum_{n=1}^{N} \frac{e^{-j\frac{4\pi}{\lambda_0}|\bar{R}_A - \bar{r}_n|}}{|\bar{R}_A - \bar{r}_n|^2} C^2(\varphi_n) e^{-j\Psi_n} \sqrt{\sigma_n}$$

The echo signal for a CW-FM radar is now derived by the aid of the results obtained hitherto.

Let the transmission signal at the antenna input be

$$U_{Tx}(t) = \cos\left[\Phi(t)\right]$$

and the course taken by the instantaneous frequency $\omega(t)$ be in accordance with Figure 8

$$\nu(t) = \frac{1}{2\pi}\frac{d\Phi}{dt} = f_0 + \left(t - \frac{T}{2}\right)\frac{\Delta f}{T} \quad \text{for } 0 < t < T.$$

Δf = frequency sweep. f_o = nominal frequency.

T = rise time.

[* Radar cross section for the given polarisation, characterised by $\vec{e}$.]

We thus have:

$$U_{Tx}(t) = \mathrm{U}\cos\left[2\pi f_0\left(t - \frac{T}{2}\right) + \pi\frac{\Delta f}{T}\left(t - \frac{T}{2}\right)^2 + \Phi_0\right] \quad (1.1.9)$$

The corresponding echo signal for a "point disperser" at point $\vec{r}_o$ is obtained, according to Eq. (8), as follows:

$$U_{Rx}(t) = \frac{\lambda_0 C^2(\varphi)\sqrt{\sigma}\ \hat{u}}{(4\pi)^{3/2}\sqrt{L_C}\ |\vec{R}_A - \vec{r}_0|^2}\cos\left[2\pi f_0\left(t - \frac{2|\vec{R}_A - \vec{r}_0|}{c} - \frac{T}{2}\right) + \right.$$

$$\left. + \pi\frac{\Delta f}{T}\left(t - \frac{2|\vec{R}_A - \vec{r}_0|}{c} - \frac{T}{2}\right)^2 + \Phi_0 - \Psi_0\right]. \quad (1.1.10)$$

See Figure 8

The echo signal is conveyed to a mixer, the LO (local oscillator) input of which is subject to a voltage $u_{LO}(t)$ which only differs from $u_{Tx}(t)$ in amplitude and possibly phase:

$$u_{LO}(t) = \hat{u}_{LO} \cos\left[2\pi f_0 \left(t - \frac{T}{2}\right) + \pi \frac{\Delta f}{T}\left(t - \frac{T}{2}\right)^2 + \Phi_1\right].$$

For NF output voltage we have

$$\tilde{u}(t) = \text{NF-Anteil}\left\{K_1\, u_{LO}(t)\, u_{Rx}(t)\right\}.$$

$$\tilde{u}(t) = \text{NF - part}\left\{K_1\ u_{LO}(t)\ u_{Rx}(t)\right\}$$

In the above K_1 is a characteristic magnitude of the mixer with the dimension 1 / V.

It follows that

$$\tilde{u}(t) = \frac{K_1\, \lambda_0\, \hat{u}\, \hat{u}_{LO}}{2(4\pi)^{3/2}\sqrt{L_C}} \frac{C^2(\varphi(\vec{r}_0))}{|\vec{R}_A - \vec{r}_0|^2} \sqrt{\sigma} \cos\left[4\pi \frac{\Delta f}{cT}|\vec{R}_A - \vec{r}_0|\left(t - \frac{T}{2}\right) + \right.$$

$$\left. + 4\pi \frac{f_0}{c}|\vec{R}_A - \vec{r}_0| + \frac{4\pi\Delta f}{Tc^2}|\vec{R}_A - \vec{r}_0|^2 + (\Phi_0 - \Phi_1) - \Psi_0\right] \qquad (1.1.11)$$

The last equation can be simplified if

(α) a filter $H_1(f)$ is built into the NF branch and is such as to balance out the distance-governed amplitude drop:

$$U(f) = H_1(f)\,\tilde{U}(f) = K_2\left(\frac{f}{f_{max}}\right)^2 \tilde{U}(f)$$

wherein $f_{max} = 2 . \frac{\Delta f}{c\,T} . R_{max}$.

We thus have, for the individual point reflectors:

$$u(t) = K_2 \frac{|\vec{R}_A - \vec{r}_0|^2}{R^2_{max}} \tilde{u}(t)$$

wherein R_{max} is an arbitrarily selectable reference distance.

(β) the preliminary factor, independent of the antenna and the antenna orientation, is abridged as follows:

$$K = \frac{K_1 K_2 \lambda_0 \, \Omega \, \Omega_{LO}}{2(4\pi)^{3/2} \sqrt{L_C} \, R^2_{max}} \quad . \qquad [K_2] = \frac{V}{m}$$

(γ) the phase term $\frac{4\pi \Delta f}{T c^2} |\vec{R}_A - \vec{r}_0|^2$ is left out of account, in relation to the other terms and the equation

$$(\delta) \; \Phi_0 - \Phi_1 = \overline{\Phi}$$

is adopted.

There remains the following:

$$u(t) = K_2 C^2(\varphi) \sqrt{\sigma} \cos\left[4\pi \frac{\Delta f}{cT} |\vec{R}_A - \vec{r}_0| \left(t - \frac{T}{2}\right) + 4\pi \frac{f_0}{c} |\vec{R}_A - \vec{r}_0| + \overline{\Phi} - \Psi \right].$$

If we now once again take a number of reflectors into account and admit dual displacements for these latter, then we have the following for the NF signal:

$$u(t) = K \sum_{n=1}^{N} C^2(\varphi(\vec{r}_n)) \sqrt{\sigma_n} \cos\left[2\pi \left(\frac{2\Delta f}{cT} |\vec{R}_A - \vec{r}_n| - f_{d,n}\right) \left(t - \frac{T}{2}\right) + \right.$$

$$\left. + 4\pi \frac{f_0}{c} |\vec{R}_A - \vec{r}_n| + \overline{\Phi} - \Psi_n \right] \quad \text{für} \quad \frac{2|\vec{R}_A - \vec{r}_n|_{max}}{c} < t < T \qquad (1.1.12)$$

The function $\varphi(\vec{r}_n)$ is to be selected in accordance with Eq. 1.1.6.

From Eq. (1.1.12) and the relation

$$f_{d,n} = \frac{2}{c} v_{r,n} \cdot f_0 \ , \quad v_{r,n} = \frac{d}{dt} |\vec{R}_A - \vec{r}_n| \ . \qquad (1.1.13)$$

for the dual displacement, it follows that a punctiform target at the distance $\rho_n \overset{def}{=} |\vec{R}_A - \vec{r}_n|$

and with a radial speed of $v_{r,n} = \frac{d\rho_n}{dt}$

will result in an NF frequency (beat frequency) of:

$$f_{\rho,n} = \frac{2\rho_n}{c} \frac{\Delta f}{T} - \frac{2}{c} v_{r,n} \cdot f_0 \qquad (1.1.14)$$

In the present example, with Δf = 120 MHz, T = 400 μs and f_o = 94 GHz, we have the following numerical value equation:

$$\frac{f_{\rho,n}}{kHz} = 2 \cdot \frac{\rho_n}{m} - 0.62\bar{6} \frac{v_{r,n}}{m/s} \qquad (1.1.15)$$

The voltage u(t) is scanned with a time increment of, for example, δt = 0.8 μs, and then provides 500 (time-range) scanning values:

$$u_\mu \overset{def}{=} u(t = \mu \delta t) \qquad \text{wherein } \mu = 0 \text{ to } 499 \qquad (1.1.16)$$

Appendix 1.2

The discrete Fourier transformation of the echo signal. Definition and approximation of resolution cell.

The discrete Fourier transformation of the order M of the scanning values u_μ of the voltage is defined via

$$U_\nu = \sum_{\mu=0}^{M-1} u_\mu \exp\left(-j\frac{2\pi}{M}\right)^{\nu \cdot \mu} \tag{1.2.1}$$

By $M = 2^Q$ the DFT can be calculated via an FFT. In the present case M may be 512 or 1024. For this purpose the six hundred scanning values are supplemented by zeroes, to give 512 or 1024 values. In order to reduce the Alias effect, moreover, a window function (filtering) can be applied. We then have: $\tilde{u}_\mu = g_\mu \cdot u_\mu$

$$\begin{aligned} \text{mit} \quad & g_\mu = \sin^2\left(\frac{\mu}{M_0}\frac{\pi}{2}\right) \text{ für } \mu = 0 \text{ bis } M_0. \\ & g_\mu = 1 \text{ für } \mu = M_0 + 1 \text{ bis } 499 - M_0 \\ & g_\mu = \sin^2\left(\frac{\mu - 499}{M_0}\frac{\pi}{2}\right) \text{ für } \mu = (499 - M_0) \text{ bis } 499 \qquad (1.2.2) \\ \text{und} \quad & g_\mu = 0 \text{ für } \mu = 500 \text{ bis } M - 1. \end{aligned}$$

In this connection M_0 is a preselectable number which should preferably be selected from the range $M_0 = 0$ to 20.

The result of the DFT for the signal, according to Eq. (1.1.12) will be derived hereunder.

For this purpose the "time gate function"

$$w_1(\xi) \overset{def}{=} \sum_{\mu=0}^{M-1} g_\mu \exp\left(-j2\pi \frac{\xi}{\delta_\rho} \frac{\mu}{M}\right) \tag{1.2.3}$$

will first of all be introduced as an auxiliary function.

Here the distance increment δ_p is given by

$$\delta_\rho = \frac{cT}{2\Delta f \cdot \delta t \cdot M} \tag{1.2.4}$$

With the system parameters T = 400 μs, Δf = 120 MHz and δt = 0.8 μs,

we thus have

$$\frac{\delta_\rho}{m} = \frac{625}{M} \tag{1.2.5}$$

For M = 512 and 1024 the distance increment thus amounts to 1.22 m and 0.67 m respectively.

The time gate function can be calculated, on the assumption M_0=0 ("no filtering with window function") and taking the given numerical values into account, as follows:

$$w_1(\xi) = \exp\left[-j2{,}5\frac{\xi}{m}\right] si\left[2{,}51 \cdot \frac{\xi}{m}\right] \tag{1.2.6}$$

$w_1(\xi)$ is thus independent of the selection of the magnitude M of the FFT.

The incorporation of Eq. (1.1.12) into Eq. (1.2.1) gives:

$$U_\nu = \frac{K}{2}\sum_{n=1}^{N} C^2(\varphi(\bar{r}_n))\sqrt{\sigma_n}\, e^{-j\Phi_n}\, w_1\left[|\bar{R}_A - \bar{r}_n| - v_{r,n} T \frac{f_0}{\Delta f} - \nu\delta_\rho\right] + \tag{1.2.7}$$

$$+ \frac{K}{2}\sum_{n=1}^{N} C^2(\varphi(\bar{r}_n))\sqrt{\sigma_n}\, e^{+j\Phi_n}\, w_1\left[|\bar{R}_A - \bar{r}_n| - v_{r,n} T \frac{f_0}{\Delta f} + \nu\delta_\rho\right].$$

wherein the phase Φ_n is calculated from the phases Ψ_n and from the distance-governed phases

$$4\pi f_0 |\vec{R}_A - \vec{r}_n| / C$$

A radial speed $v_{r\,n}$ of the n^{th} punctiform target has the same effect, where the signal U_v is concerned, as a displacement of the distance $|\vec{R}_A - \vec{r}_n|$ by $\frac{\Delta}{m} = 0.313 \frac{v_{r,n}}{m/s}$.

In the calculation of U_v for "high" positive v values the second term in Eq. (1.2.7) can be neglected. If, in addition, the double displacement is negligible, the following remains:

$$U_v \approx \frac{K}{2} \sum_{n=1}^{N} C^2(\varphi(\vec{r}_n))\, w_1\left[|\vec{R}_A - \vec{r}_n| - v\delta_\rho \right] \sqrt{\sigma_n}\, e^{-j\Phi_n}$$

$$= \sum_{n=1}^{N} w(\vec{r}_n) \sqrt{\sigma_n}\, e^{-j\Phi_n} \tag{1.2.8}$$

Eq. (1.2.8) shows that U_v is the weighted sum of all complex-value scatter amounts $\sqrt{\sigma_n} \exp(-j\,\Phi_n)$. The weight function $w(\vec{r}_n)$ is found to be:

$$w(\vec{r}_n) = \frac{K}{2} w_1\left[|\vec{R}_A - \vec{r}_n| - v\delta_\rho \right] C^2(\varphi(\vec{r}_n)) \tag{1.2.9}$$

and thus results from the n time gate function w_1 and gain function C^2 of the antenna.

$w(\vec{r})$ is at its maximum when $|\vec{R}_A - \vec{r}_n| = v\delta_\rho$ and $\varphi = 0$, i.e. at the point

$$\vec{r}_z = \vec{R}_A + \vec{u}_l \cdot \nu \delta_\rho . \qquad (1.2.10)$$

This point, which depends on the orientation $\vec{u_l}$ of the antenna lobe and on the number ν of the distance gate, is designated the central point of the resolution cell.

By the aid of a calibration measurement (see Sect. 2.3) and suitable scaling of the voltage U_ν it is possible to ensure that

$$w(\vec{r}_z) = 1 \qquad (1.2.11)$$

applies.

Taking into account the fact that the function $w(\vec{r})$ is negligible for points further than 4 m from $\vec{r_z}$, it is possible to introduce, for $|\vec{R}_A - \vec{r}_n| - \nu \delta_\rho$ die , the longitudinal distance

$$\zeta_n = | \vec{u}_l \, (\vec{r}_n - \vec{r}_z) |$$

as an approximation. The longitudinal weight function thus becomes

$$w_l \approx si\left[2.51 \cdot \frac{\zeta_n}{m} \right].$$

This si function can be approximated, for arguments less than π, by a Gauss function

$$w_l \approx \exp\left[-\left(\frac{\zeta}{\zeta_0} \right)^2 \right] \qquad (1.2.12)$$

with $\zeta_0 \approx 0.92$ m.

See Figure 9.

The transversal weight function derived from the gain function can be indicated by the aid of the transversal distance (see also Eq. 1.1.5):

$$\xi = \sqrt{|\vec{r} - \vec{r}_Z|^2 - \zeta^2}$$

$$w_{tr}(\xi) = \frac{K}{2} C^2(\varphi(\vec{r})).$$

If this function is approximated by a Gauss function, then taking into account an average distance Ro of the radar target from the phase centre of the antenna and a half-width value of 2.1 degrees for the antenna, we have:

$$w_{tr}(\xi) \approx \exp\left[-\left(\frac{\xi}{\xi_0}\right)^2\right]$$

$$\xi_0 = R_0 \cdot \frac{2{,}1 \cdot \pi}{2 \cdot 180} \cdot 1{,}2 \approx 0{,}022 \cdot R_0. \qquad (1.2.13)$$

For Ro $\approx$ 115 m, therefore, we have $\xi_0 \approx$ 2.5 m.

Finally, by combining the transversal and the longitudinal weight function, we have:

$$\begin{aligned} w &= w_{tr} \cdot w_l \\ &\approx \exp\left\{-\left[\left(\frac{\xi}{\xi_0}\right)^2 + \left(\frac{\zeta}{\zeta_0}\right)^2\right]\right\} \end{aligned} \qquad (1.2.14)$$

Appendix 1.3

Discussion of effective radar cross section for resolution cell. Introduction of statistical description.

According to Eq. (1.2.8) from appendix 1.2 the values U_v available after the FFT are indicated by the formula

$$U_v = \sum_{n=1}^{N} w(\vec{r}_n)\, e^{-j\Phi_n} \sqrt{\sigma_n} \tag{1.3.1}$$

U_v is thus the sum, weighted by $w(\vec{r}_n)$, of the complex-value individual scatter magnitudes $e^{-j\Phi_n}\sqrt{\sigma_n}$.

For $|U_v|^2$ -- this is the "effective radar cross section" for the resolution cell selected - it follows that:

$$|U_v|^2 = \sum_{n=1}^{N} \sum_{m=1}^{N} w(\vec{r}_n)\, w(\vec{r}_m) \sqrt{\sigma_n} \sqrt{\sigma_m}\, e^{-j(\Phi_n - \Phi_m)} \tag{1.3.2}$$

In the discussion of these results the influence of the positively real function ($w(\vec{r}$) and the phases $\Phi_n - \Phi_m$ should preferably be considered separately.

The different spatial weightings of the individual magnitudes according to the function $w(\vec{r})$ is the result of the directional effect of the antenna and of the distance gate function. If the location of the phase centre of the antenna $\vec{H}_A$ is considered to be fixed and if the antenna lobe is then tilted and/or the

distance gate parameter altered, then the position of the central point of the resolution cell will change. In this process only the function $w(\vec{r})$, but not the phases Φ_n, will change in Eq. (1.3.1) and (1.3.2). As $w(\vec{r})$ is a function which only "changes slowly" with the location of (3 dB - width > 1 m) Uv and $|Uv|^2$ (effective radar cross section), under the said conditions, will likewise only be slowly changing functions of the central point of the resolution cell. With a fixed locality for the centre of the phase and with a fixed radar target, therefore, the dependence of the effective radar cross section $|Uv|^2$ on the position of the resolution cell can be defined by a few parameters.

Let us now consider the case in which $w(\vec{r})$ remains unchanged (fixed resolution cell) while the locality $\vec{R}_A$ of the phase centre changes. As the phases Φ_n have an additive magnitude, dependent on the distance between the phase centre $\vec{R}_A$ and the location of the "scatter centre" $\vec{r}_n$, we have the following in Eq. (1.3.2).

$$\Phi_n - \Phi_m = \Psi_0 + \frac{4\pi f_0}{c}\left[|\vec{R}_A - \vec{r}_n| - |\vec{R}_A - \vec{r}_m|\right] \tag{1.3.3}$$

$$\begin{aligned}\Phi_n - \Phi_m \approx \Psi_0 + \frac{4\pi f_0}{c}\Big[(\vec{r}_m - \vec{r}_n)\cdot\vec{u}_r + \frac{1}{2}\left((\vec{r}_n)^2 - (\vec{r}_n\cdot\vec{u}_r)^2\right)\frac{1}{R} + \\ + \frac{1}{2}\left((\vec{r}_m)^2 - (\vec{r}_m\cdot\vec{u}_r)^2\right)\frac{1}{R}\Big].\end{aligned} \tag{1.3.4}$$

wherein $\vec{u}_r$ is the unit vector and R the distance of $\vec{R}_A$ to the central point of the resolution cell.

The phase differences $\Phi_n - \Phi_m$ are dependent on $\vec{R}_A$. If we consider two dispersion (scatter) magnitudes of which the locations are transverse to $\vec{u}_r$ and separated by the distance D, we see that if the aspect angle changes by

$$\delta\alpha / \text{DEGREES} \approx 15 \cdot \frac{\lambda_D}{D} \qquad (1.3.5)$$

(R fixed) there is a change from constructive to destructive interference.

If we make D equal to the diameter of the resolution cell, i.e. about 2 m, $\lambda_0 =$ 3.2 mm, then it follows that $\delta\alpha \approx 1/40$ degrees.

The effective radar cross section $|U_v|^2$ is thus a very fast-changing function of the aspect direction.

As long as R does not exceed $2\,D^2/\lambda_0$ the second term in Eq. (1.3.4) cannot be neglected. $|U_v|^2$ is thus also dependent on R. For the above numerical values, R must be greater than 2.5 Km, so that $|U_v|^2$ is independent of R, i.e. the radar signal will be within the scatter field of the illuminated part of the radar target.

From the details given hitherto it can be seen that the description of the dependence of $|U_v|^2$ on the

location of the phase centre necessitates a very large number of parameters. A statistical model is introduced in its place for the back scatter and is in each case valid for a preselectable aspect angle range (as a typical case, $\Delta\alpha = 3°$).

Let the overall volume be considered to be subdivided into volume ranges and a coherent and non-coherent scatter magnitude $\sqrt{\sigma_i^K}$ and $\sqrt{\sigma_i^{NK}}$ respectively be associated with each part-range (centre of range: r_i).

We then have the following -- according to Eq. (1,3,2) for a fixed aspect direction.

$$|U_v|^2 = \sum_i w^2(\bar{r}_i)\,\sigma_i^{NK} + \sum_i\sum_j w(\bar{r}_i)\,w(\bar{r}_j)\sqrt{\sigma_i^K}\,\sqrt{\sigma_j^K}\;e^{j(\Phi_i-\Phi_j)} \qquad (1.3.6)$$

If we now assume that

(α) σ_i^{NK}, within the selected aspect angle range is independent of the aspect angle,

(β) the variation of $\sqrt{\sigma_i^K}$ and $\sqrt{\sigma_j^K}$ with the aspect angle within the preselected range is as follows:

$$p\left(\sqrt{\sigma_i^K}\right) = \frac{2\sqrt{\sigma_i^K}}{\langle\sigma_i^K\rangle}\exp\left(-\frac{\sigma_i^K}{\langle\sigma_i^K\rangle}\right)$$

and

$$p\left(\sqrt{\sigma_j^K}\right) = \frac{2\sqrt{\sigma_j^K}}{\langle\sigma_j^K\rangle}\exp\left(-\frac{\sigma_j^K}{\langle\sigma_j^K\rangle}\right)$$

(angular brackets " $\langle\ \rangle$ " denote average value

(γ) $p(\Phi_j) = \frac{1}{2\pi}$ und $p(\Phi_i) = \frac{1}{2\pi}$

and

(δ) the variation of the different scatter magnitudes is statistically independent, then we have for the average value of $|Uv|^2$:

$$\langle |U_v|^2 \rangle = \sum w^2(\bar{r}_i)\left(\langle \sigma_i^{NK} \rangle + \langle \sigma_i^K \rangle\right) \qquad (1.3.7)$$

and the variance

$$\langle \left(|U_v|^2 - \langle |U_v|^2 \rangle\right)^2 \rangle = \left(\sum_i w^2(\bar{r}_i)\langle \sigma_i^K \rangle\right)^2 \qquad (1.3.8)$$

The distribution function for $|Uv|^2$, i.e. for the radar cross section associated with a fixed position of the resolution cell and a fixed aspect angle range, is thus

$$p\left(|U_v|^2\right) = \frac{1}{A} \exp\left(-\frac{|U_v|^2 - B}{A}\right) \qquad (1.3.9)$$

wherein $A = \sum w^2 \langle \sigma_i^K \rangle$

and $B = \sum w^2 \langle \sigma_i^{NK} \rangle$.

Appendix 1.4

Description of Model Used:

The backscatter of the radar target is modelled by I punctiform disperser (nodal radar cross sections). The data of the individual "punctiform dispersers" in each case only apply to a fixed aspect angle range ($\Delta\alpha = 3°$) in the azimuth) and thus change with this range. There are a total of 360/3 = 120 different aspect angle ranges (or 60, if the radar target has a plane of symmetry).

Within an aspect angle range the radar cross section σ_i of the i^{th} punctiform disperser is described by superimposition of a coherent part σ_i^K and an incoherent part σ_i^{NK}. The variation with the aspect angle (in the azimuth within the range of 3° and with an adequate variation of the elevation angle) is indicated by the distribution function:

$$p(\sigma_i) = \frac{1}{\langle\sigma_i^K\rangle} \exp\left[- \frac{\sigma_i - \sigma_i^{NK}}{\langle\sigma_i^K\rangle} \right]$$

The scatter of a punctiform disperser for an aspect angle range is thus completely described by two real parameters, i.e. the average of the coherent part $\langle\sigma_i^K\rangle$ and the incoherent part σ_i^{NK}. It should be noted that the variance of σ_i with $\langle\sigma_i^K\rangle$ is related in accordance with the equation:

$$\langle(\sigma_i - \langle\sigma_i\rangle)^2\rangle = \langle\sigma_i^K\rangle^2 \qquad (1.4.2)$$

The complete description of the backscatter over all aspect angle ranges thus requires I x 120 x 2 real parameters. This number is halved in the event of a plane of symmetry.

From these parameters and for the scanning value U_v of the voltage in the frequency range associated with a fixed aspect angle and a fixed position of the resolution cell, i.e. the scanning value introduced in appendix 1.2, it is possible to calculate the distribution function in the following form:

$$p(|U_v|^2) = \frac{1}{A} \exp\left(-\frac{|U_v|^2 - B}{A}\right) \qquad (1.4.3)$$

$$A = \sum_{i=1}^{I} w^2(\vec{r}_i - \vec{r}_{\zeta\nu}) \langle \sigma_i^{K} \rangle \qquad (1.4.4)$$

$$B = \sum_{i=1}^{I} w^2(\vec{r}_i - \vec{r}_{\zeta\nu}) \langle \sigma_i^{NK} \rangle \qquad (1.4.5)$$

In the above $\vec{r}_{\zeta\nu}$ is the central point of the resolution cell and $w^2(\vec{r}_i - \vec{r}_{\zeta})$ is the weight function defined according to Appendix 1.2 (directional effect of the antenna and time gate function). It should be noted that the average value of $|U_v|^2$ is indicated by

$$\langle |U_v|^2 \rangle = A + B \qquad (1.4.6)$$

and the variance by:

$$\langle (|U_v|^2 - \langle |U_v|^2 \rangle)^2 \rangle = A^2$$

Appendix 2

Continuous calculation of average value and of variance:

Calculation of average:

$$\bar{x}_N \stackrel{def}{=} \frac{1}{N} \sum_{n=1}^{N} x_n$$

It thus follows, for N + 1:

$$\bar{x}_{N+1} = \frac{1}{N+1} \sum_{n=1}^{N+1} x_n = \frac{1}{N+1} \left(\sum_{n=1}^{N} x_n + x_{N+1} \right)$$

(A. 1)

$$\bar{x}_{N+1} = \frac{1}{N+1} \left(N \cdot \bar{x}_N + x_{N+1} \right)$$

Calculation of variance:

$$S_N \stackrel{def}{=} \frac{1}{N} \sum_{n=1}^{N} (x_n - \bar{x}_N)^2$$

$$= \frac{1}{N} \left(\sum_{n=1}^{N} x_n^2 - 2\bar{x}_N \underbrace{\sum_{n=1}^{N} x_n}_{N \cdot \bar{x}_N} + \underbrace{\sum_{n=1}^{N} \bar{x}_N^2}_{N \cdot \bar{x}_N^2} \right)$$

$$S_N = \frac{1}{N} \sum_{n=1}^{N} x_n^2 - \bar{x}_N^2$$

It thus follows, for N + 1:

$$S_{N+1} = \frac{1}{N+1} \sum_{n=1}^{N+1} x_n^2 - \bar{x}_{N+1}^2$$

(A. 2)

$$S_{N+1} = \frac{1}{N+1} \left[N \left(S_N + \bar{x}_N^2 \right) + \bar{x}_{N+1}^2 \right] - \bar{x}_{N+1}^2$$

Appendix 3

Calibration Measurement:

The model to be calculated later for the radar backscatter is intended to provide a correct description of the absolute value of the radar cross section (average and variance for a given position of the resolution cell).

For this purpose a measurement must be carried out in advance on a radar target (calibration radar target)

(α) of which the (far-zone) radar cross section σ_{ok} is known,

and

(β) for which the antenna is situated in the dispersion far zone.

To fulfil the condition (β) the maximum geometrical transverse dimension of the calibration radar target D_K must be not greater than

$$D_{K.max} = \sqrt{\frac{\lambda_0 R_0}{2}}$$

In the above, R_o is the oblique distance between radar target and antenna. With R_o = 115 m and λ_o = 0.32 cm, we have $D_{K\ max}$ = 43 cm.

When the antenna lobe is pivoted and when the various distance gates are covered (numbering ν) the central point of the resolution cell $\vec{rz}$ assumes different

discrete local positions (see Eq. (1,2,10).

In the calibration measurement, taking account of the extent of the resolution cell, it is necessary to ensure that the value $|U_\nu|^2 = U_0^2$ (scanning values in the frequency range) associated with the target is that at which the central point $\vec{rz}$ of the relevant resolution cell is not distant by over about 30 cm in the radiation direction or by over about 60 cm transversally to the radiation direction from the main scatter centre of the radar target. With an angle reflector this scatter centre is situated at the apex.

In order to fulfil this condition the measurement is repeated with different positions of the calibration radar target and the maximum value $U_0^2 = \text{Max}\left[|U_\nu|^2\right]$ is stored. If we assume that in Eq. (1.2.8) the voltage U_ν is m scaled to $U_0/\sqrt{\sigma_{oK}}$ for the extended radar target to be examined, then the weight function introduced there will be in accordance with Eq. (1.2.11).

Within the framework of the calculation of the parameters of the backscatter model the scaling of the data can be carried out in the calculation of the local radar cross sections.

CLAIMS

1. Radar antenna arrangement for determining the effective radar echo cross section of radar targets as a function of the geometrical position of the target in relation to the radar antenna arrangement, wherein:-

(a) the radar antenna is mounted on a rotatable platform outside the rotation axis of the platform;

(b) the radar target to be examined is located on a rotating stage; and

(c) the platform is pivotable about two axes perpendicular to the rotation axis, so that the entire radar target can be successively scanned over a period of time.

2. Radar antenna arrangement in accordance with Claim 1, wherein the axis of the direction of maximum radiation of the antenna is inclined in relation to the rotation axis of the platform, the relevant angle of inclination being selected to ensure that in the periods in which the position of the resolution cell in relation to the radar target only changes negligibly, the aspect angle from which the phase centre of the radar antenna is viewed from the radar target passes through a preselected angular range in respect of the azimuth of the elevation.

3. Radar arrangement in accordance with one of Claims 1 or 2, wherein the operation of determining the effective radar backscatter cross section is effected using a CW-FM radar apparatus or a pulse doppler radar apparatus.

4. Method for determining the effective radar backscatter cross section of a target using the radar arrangement according to any one of Claims 1 to 3, wherein:

(a) the time dependent radar return signals are measured as a function of the position of the platform and the position of the rotating stage and subjected to a Fourier transformation;

(b) from the (discrete) spatial position of the resolution cell in each case and from the return signal data for an aspect angle range in each case the average value and the variance are determined;

(c) by allocating the average values and variances to the centre point of the resolution cell for each aspect angle range a three-dimensional image function of the average values and a three-dimensional image function of the variances are formed and are allocated to preselected geometrical positions for the scattering centres in order to indicate the backscatter characteristics of the

target;

(d) the punctiform local radar cross sections are described for each aspect angle range by the average value of the radar cross section and its variance;

(e) from the average value and variance indications of the local radar cross sections and by using suitable weighting the average value and the variance of the effective radar cross section for a given resolution cell and a given aspect angle range are determined.

5. A radar arrangement constructed and arranged to function substantially as herein described and illustrated and as exemplified.

6. A method of determining radar backscatter cross section as herein described and exemplified.

Patents Act 1977 **Examiner's report to the Comptroller under Section 17 (The Search Report)**	Application number 9124586.0
Relevant Technical fields (i) UK Cl (Edition K) H4D (DMXX,DMSS,DX) (ii) Int Cl (Edition 5) G01S **Databases** (see over) (i) UK Patent Office (ii) ONLINE DATABASES: WPI, INSPEC, EDOC	Search Examiner DR E P PLUMMER Date of Search 26 MARCH 1992

Documents considered relevant following a search in respect of claims ALL

Category (see over)	Identity of document and relevant passages	Relevant to claim(s)
A	Flight International 7 September 1985 pages 21, 22	
A	Proc. 1988 IEEE National Radar Conference (Cat. No. 88CH2572-6) pages 209-13 pub.IEEE New York:Scheer et al: MMW radar cross section range characterizes targets	

Category	Identity of document and relevant passages	Relevant to claim(s)

Categories of documents

X: Document indicating lack of novelty or of inventive step.

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